Microwave Journal Expert Advice - Jan 09

Forks in the Road of Antenna Analysis and Design

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"Simplify, simplify" Henry David Thoreau

1. Introduction

Legend has it that the inventor of the Beverage Antenna, Harold Henry Beverage (1893-1993) was in the audience at a lecture concerning the mathematical modeling of that antenna. At the conclusion, he remarked that, "Had I known this antenna was so complicated, I never would have invented it!"

In my own firsthand experience, circa 1975, a discussion with a senior DoD program manager about military telecommunications seared a permanent record into my memory. When the talk got around to antennas, he glossed over their importance with the remark, "We just drop a wire outside the porthole, and then we crank in all the signal processing gain we need to maintain the communications link." This manager obviously had no clear picture of how antennas worked or how critical they were for wireless communications.

In a rare concession to diplomacy, at least for me, I held my tongue and weakly smiled. Keeping my thoughts completely to myself, I was confident that the folly of this attitude would become apparent within a few short years, as the performance of complex systems fell short of expectations. Sooner or later, the necessity for of grabbing as much incoming signal power as possible with good antennas at the very front end of communications systems should become glaringly obvious.

I also asked myself, "What simple words can I say to this influential manager so that he can appreciate the importance of a good antenna to a telecommunications system?" Alas, the operational word was "simple", and I had no answer because antenna analysis and design was a complicated and mathematically intensive business.

Unfortunately today, that program manager's philosophy prevails. Too often, antennas must endure serious compromises to accommodate other design constraints of telecommunications devices and systems. For example, they must be tiny compared with a quarter wavelength, challenging their efficiency. They must be etched onto printed circuit boards or otherwise be located in close proximity to conducting bodies, challenging their directive gain.

Part of the problem is the way antenna design and analysis has evolved. Of all the disciplines within electromagnetics and electronics, antenna theory is near the top when it comes to mathematical complexity and massive numerical simulations. This has caused at least two major problems. First, because of complexity, we might be missing some important design options. Second, it is difficult to explain in simple terms, and otherwise sell, the project managers on the essential role and minimum design requirements of antennas.

Even in the rarefied atmosphere of academia, antennas have a certain aura of mystery borne out of mathematical complexity. When I was a graduate student and confided a special interest in antennas to my faculty advisor, he exclaimed, "Omigod! You mean you actually understand how those crazy electrons race around on those wires?"

Does it have to be this way? Perhaps antenna theory has dug itself into a hole, making it so specialized and difficult that only an antenna theorist could love it. As engineering disciplines evolve, there are many forks in the road. It might be that antenna theory just so happened to take the forks that led down a path of increasing mathematical complexity.

What might have happened if antenna theory had taken another path, one that led to simplification instead of increasing mathematical sophistication? This column will explore one such "what if". The example comes from the world of wire antennas; however, thanks to the Theory of Complementary Antennas, it also embraces small apertures. Such stories can also be told about the world of large apertures.

2. A Very Brief History of Antenna Analysis

As prelude to our hypothesis, we recall the greatest simplification of all in electromagnetics, the Maxwell Equations. James Clerk Maxwell did not publish them in any form we might recognize today. There were not four vector equations, but no less than 20 scalar ones. They were a testimony to the genius of Maxwell for quantifying an important branch of physics; however, mathematically they were intractable.

It is difficult to imagine where electromagnetics and antennas might be today had it not been for the simplifying insights of Oliver Heaviside. Improvising vector notation, he distilled the 20 Maxwell Equations down to the four we use today.

How might antenna analysis and design be different today if there were a fundamental simplification in the tradition of Heaviside?   We can answer that question, at least in part, by briefly reviewing the course of antenna theory.

Detailed mathematical models of antennas got their start in 1897, with a published paper by Henry Cabourn Pocklington (1870-1952) [1]. He derived an integral equation for the current on a straight wire of length $2L$ and radius a,

$$\int_{-L}^{L} I(u)\left(\frac{\partial^2}{\partial z^2}+k^2\right)G(u,z)du=-j\omega\varepsilon_0 E(z) \tag{1}$$

In eqn. 1, k is the wavenumber, $E(z)$ is the parallel component of the electric field vector at the surfaced of the wire, and the free space Green's function is:

$$G(u,z)=\frac{e^{-jk\sqrt{a^2+(z-u)^2}}}{4\pi\sqrt{a^2+(z-u)^2}} \tag{2}$$

Eqn. 1 was further constrained by a boundary condition at the ends of the wire, namely, that the current must vanish because it had no place to go:

$$I(L)=I(-L)=0 \tag{3}$$

Arguing that the right side of eqn. 1 was zero except between the terminals of the antenna, Pocklington deduced that the solution to the integral equation that also satisfied the boundary condition must be:

$$I(z)=I_0 \sin k\left(L-|z|\right) \tag{4}$$

Pocklington's analysis largely determined the course of antenna theory for about 75 years. Actually, he determined two competing courses, but they progressed in parallel. One was the pursuit of increasingly sophisticated solutions to the integral equation, eqn. 1. The other was the pursuit of transmission line analogies based on sinusoidal current distributions similar to eqn. 4.

Pocklington may have also set the mathematically intensive tone for antenna analysis. Recognized as a mathematician, a physicist, and an astronomer, he is mostly remembered for his contributions to number theory, notably Pocklington's Theorem for Prime Numbers.

In the United States, the most prolific proponent of the integral equation approach was Prof. Ronold W. P. King (1905-2006) of Harvard University [2]. His counterpart for transmission line analogies was Sergei A. Schelkunoff (1896-1992) of Bell Laboratories [3]. The two vigorously debated the relative merits of their approaches for decades until about 1970.

Circa 1970, antenna analysis and design came to an important fork in the road, marked by the ready availability of digital computers. Integral equations are computer friendly because numerical integration tends to smooth over errors. Further, the integral equation approach was extended and generalized by a mathematical procedure called the Method of Moments [4], which was also computer friendly. As a result, many intractable problems in antenna analysis and electromagnetics that had accumulated for decades suddenly became practical, if not actually easy, to solve.

So, the integral equation approach and transmission analogies parted ways. The former dominates antenna analysis and design today. The latter is seldom mentioned.

But the price of generality is complexity, and this is certainly true when it comes to applying computer simulations to antenna analysis and design. If we want to argue the merits of a particular design to a project leader, then we must come armed with a box load of computer printout.

Might things have turned out differently?

3. The Fork That Wasn't Taken

There was a major simplifying opportunity, circa 1941, that was missed. It concerned characteristic impedance. Both the integral equation school and the transmission line analogy school came up with formulas for characteristic impedance; however, the formulas were different and it was difficult to understand a general physical interpretation for them. The integral equation version is,

$$Z_0 = \frac{\eta}{2\pi}\log\left(\frac{2L}{a}\right) \tag{5}$$

The transmission line analogy version is,

$$Z_0 = \frac{\eta}{2\pi}\left[\log\left(\frac{2L}{a}\right) - 1\right] \tag{6}$$

In eqns. 5 and 6, η is the impedance of free space.

It is seen that the equations are similar but do not agree exactly. Further, there is a disturbing dependence upon the length of the antenna *L* in both formulas. In contrast, in formulas for the characteristic impedance of waveguides and transmission lines, there is no such dependence. Characteristic impedance depends only upon cross sectional geometry, not the length of the device. These problems prevented further pursuit of characteristic impedance as a fundamental antenna design parameter. It remained simply a mathematical result that bore an intriguing but coincidental resemblance to the characteristic impedance of transmission lines.

This was an important, if not critical, fork in the road of antenna design and analysis. If the misgivings about the formulas for characteristic impedance had been resolved and pursued, then antenna analysis and design would have been greatly simplified. For one, the model of the basic dipole itself would have been simplified as demonstrated in the article in the January, 2009, issue of the Microwave Journal.

In a recent book [5], the author revisited this fork in the road and derived a more general formula for characteristic impedance:

$$\boxed{Z_0 = -\frac{\eta}{2\pi}\ln(k'a)} \tag{7}$$

In eqn. 7, k' is the quasi static wave number:

$$k' = \frac{e^{\gamma}}{2}k \tag{8}$$

In eqn. 8, γ is Euler's constant (0.5772156649…)

It is seen that the chief objection to eqns. 5 and 6 have been removed. There is no dependence upon the length of the antenna. Only the cross sectional area matters.

Now that a general formula for antenna characteristic impedance is available, it makes sense to define and use a common mode voltage along the antenna,

$$V(z) = Z_0\left(I_F e^{-jkz} - I_R e^{jkz}\right) \tag{9}$$

In eqn. 9, I_F and I_R are the complex amplitudes of the forward and traveling waves of current along the antenna. With a well defined voltage, it is now very easy to model discontinuities in antenna geometry, antenna folds, and insertions of lumped loads and transmission line stubs.

4. Complicated Antennas Made Simple

With a general formula for characteristic impedance, countless antennas and antennas immersed in complicated environments become mathematically more tractable. Even more important, physical insight is less obscured, promoting design innovations. We'll mention just a few examples here.

Two families of antennas that have benefited greatly from eqn. 7, with respect to analysis and design are loaded antennas and folded antennas. Using the general formula for characteristic impedance, mathematical models for these antennas have been reduced to simple formulas using little more than algebra. Performance predicted by these models agrees very closely, if not exactly, with that obtained experimentally and/or using much more complicated mathematical and numerically intensive approaches.

For a detailed discussion of the loaded antenna and its simplified mathematical model, Download Raines2. For the folded antenna, Download Raines3.

Another family, pervasive in today's designs, is the printed circuit antenna. Today, they are the subject of specialized analyses and text books; however, in many cases, they may be regarded simply as wire antennas with an appropriate change in characteristic impedance. The dielectric circuit board increases the capacitance per unit length *C* of that characteristic impedance, and so it decreases with respect to a wire in free space, according to the formula,

$$Z_0 = \sqrt{\frac{L}{C}} \tag{10}$$

Further, that same capacitance per unit length increases the wave number, according to the formula,

$$k = \omega\sqrt{LC} \tag{11}$$

This increase in wave number means that the antenna may be slightly shorter than the equivalent wire in free space. Equivalently, the effective length of the printed circuit board antenna increases compared with a wire in free space.

On crowded circuit boards, one problem is the interaction of the antenna with adjacent strips of conductor. These, too, can be regarded as wires in free space with an appropriate decrease in characteristic impedance and increase in longitudinal dimensions. With those simplifications, the printed circuit board becomes an antenna array with parasitic elements.

Cladded antennas are yet another family that was popular in the literature some years ago; however, again the treatment was specialized and mathematically intensive. With a general formula for characteristic impedance, however, cladded antennas are simply related to wires in free space.

If the antenna is cladded with a dielectric, then the capacitance per unit length increases, and the characteristic impedance decreases, according to eqn. 10. If the antenna is cladded with a ferrite, then the inductance per unit length L increases, and the characteristic impedance increases. Thus, cladding is a good way to adjust the characteristic impedance, and thus the input impedance, of antennas.

5. Conclusions

Traditions, once entrenched, die hard and prolonged deaths. This is true not only for cultural phenomena but for mathematical models of physical phenomena as well. Antenna theory and design is a prime example. Mathematically intensive from the very beginning in the late Nineteenth Century, and now intensive with respect to numerical simulations, some simplifying points of view have been overlooked. As a result, an unknown number of design innovations have certainly fallen by the wayside.

We can draw encouragement for a simplifying point of view from three different celebrities, a physicist, an economist, and an artist. Albert Einstein is famous for the quote, “Everything should be made as simple as possible, but not simpler.”

Economist E. F. Schumacher is known for saying, “Any intelligent fool can make things bigger and more complicated. It takes a touch of genius---and a lot of courage--- to move in the opposite direction.”

Finally, artist Hans Hofmann observed that, “The ability to simplify means eliminating the unnecessary so that the necessary can speak.”

References

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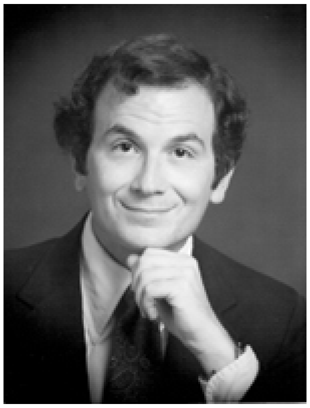

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