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Simple Formula Relating dBm and dBu

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1. Introduction

Standards and specifications for electronic devices, especially those used in wireless communications, are often expressed in dissimilar terms, either dBm (decibels above one milliwatt) or dBu (decibels above one microvolt per meter). For example, the boundary of the CGSA (Cellular Geographic Service Area) is defined by the FCC to be that distance at which a 32 dBu electric field intensity is broadcast. On the other hand, the sensitivity of a cell phone is usually described in terms of dBm, typically on the order of -110 dBm.

At first glance, it might appear that these two units are unrelated to each other, or at least not simply related. But they are. In this brief note, we will derive that relationship. The resulting formula is simple, requiring no more than a little algebra to evaluate. It is a relationship that should be at every engineer's fingertips; however, it is often our bad luck that, just when we need some quick answers, the formula is usually misplaced or buried in some reference where we cannot conveniently find it.

We will find such a relationship essential for answering practical questions such as, Can a cell phone with sensitivity of -115 dBm be expected to operate in a rural region with only 20 dBu coverage from a distant base station? We will return to this question after deriving the formula.

2. Derivation of Formula

Fig. 1 shows the configuration of interest. An electromagnetic plane wave from a distant source is incident upon an electronic device. The power received by the latter is:

$$P = SA \qquad (1)$$

In eqn. 1, S is the power density incident upon the device in watts per square meter, and A is the effective area of the device as an antenna in square meters.

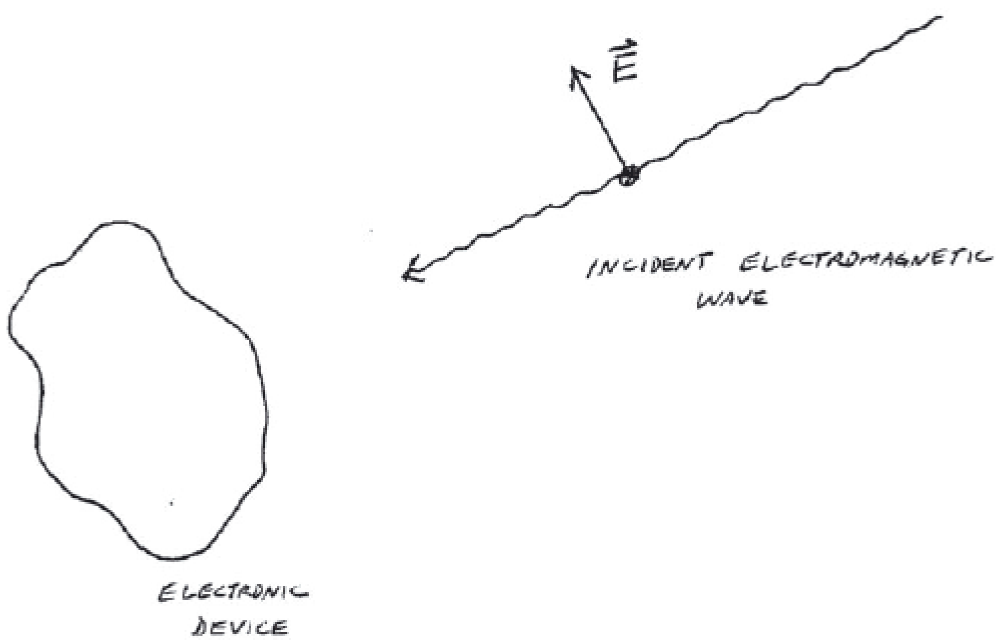

Figure 1 – An electromagnetic plane wave is incident upon an electronic device. The intensity of the plane wave is expressed as dBu, while the sensitivity of the device is expressed as dBm. Given those metrics, how do they interact?

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For an electromagnetic plane wave, the power density is:

$$S = \frac{E^2}{\eta} \tag{2}$$

where E is the electric field intensity in volts per meter, and η is the impedance of free space (about 377 ohms).

For an antenna, or for any device acting as an antenna, the effective area is:

$$A = \frac{\lambda^2}{4\pi} g \tag{3}$$

Where λ is the wavelength, and g is the directive gain of the antenna.

Combining equations 1 through 3, we obtain a simple relationship between the electric field incident upon the device, and the power received by the device:

$$P = E^2 \frac{\lambda^2 g}{4\pi\eta} \tag{4}$$

Equation 4 is in linear form; however, standards and specifications are usually expressed in logarithmic form. To convert from one form to the other, we simply take the logarithms of both sides:

$$\begin{aligned} 10 \log P = 20 \log E + 10 \log(\lambda^2 g) \\ - 10 \log(4\pi\eta) \end{aligned} \tag{5}$$

To express equation 5 in terms of the popular units milliwatts and microvolts per meter, we note the relations:

$$P = mW \times 10^{-3} \tag{6}$$

And

$$E = \mu V/m \times 10^{-6} \tag{7}$$

Combining equations 5 through 7, we obtain:

$$\begin{aligned} 10 \log(mW) = 20 \log(\mu V/m) \\ + 10 \log(\lambda^2 g) - 126.76 \end{aligned} \tag{8}$$

In equation. 8, the left side is the expression for dBm, and the first term on the right side is the expression for dBu. So, the simple formula that we set out to derive is:

$$\boxed{dBm = dBu + 10 \log(\lambda^2 g) - 126.76} \tag{9}$$

In equation 9, as we have already noted, λ is the wavelength in meters, and g is the directive gain of the electronic device acting as an antenna.

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Some useful values for the directive gain g are: 1) 1.5 for a short dipole, and 2) 1.64 for a half-wave dipole.

Fig. 2 is a plot of equation 9 for a wide range of dBu and frequencies, and for a short dipole.

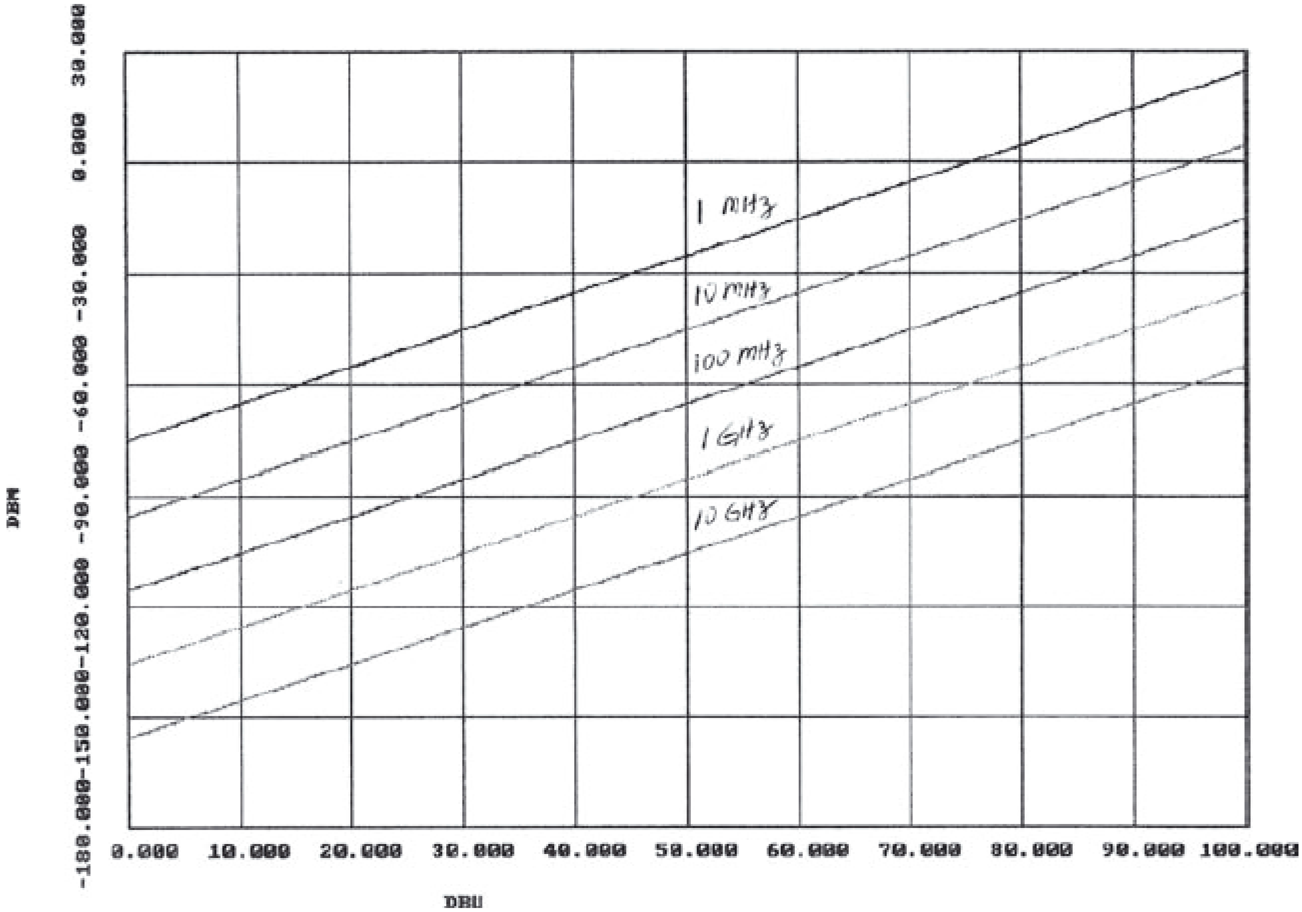

Figure 2 – dBm as a function of dBu for a short dipole and a wide range of frequencies

3. Conclusions

We have a derived a simple formula relating the sensitivity of electronic devices to an incident electromagnetic field. It depends upon the frequency or wavelength and the directive gain of the device. That directive gain may be deliberately engineered, for example, in the case of a cell phone antenna. It may also be unintentional in cases of electromagnetic interference.

We return to the question we posed at the beginning of this note: Can a cell phone with a sensitivity of –115 dBm be expected to operate in an area of weak coverage providing only 20 dBu of signal? Let's use equation 9 to find the answer. We'll use the worst case of 1900 MHz, or a wavelength of 0.158 meters, and a short dipole with gain of 1.5.

$$dBm = 20 + 10\log(1.5 \times 0.158^2) - 126.76 = -121 \qquad (10)$$

The answer to our question is: No, the cell phone cannot be relied upon in this remote area. It will require a high gain antenna and/or a signal booster.

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Biography

Jeremy Keith Raines received his BS degree in electrical science and engineering from MIT, his MS degree in applied physics from Harvard University and his Ph.D. degree in electromagnetics from MIT. He is a registered professional engineer in the state of Maryland. Since 1972, he has been a consulting engineer in electromagnetics. Antennas designed by him span the spectrum from ELF through SHF, and they may be found on satellites deep in space, on ships, on submarines, on aircraft, and at a variety of terrestrial sites. Dr. Raines is a senior member of IEEE. He may be contacted at www.rainesengineering.com.